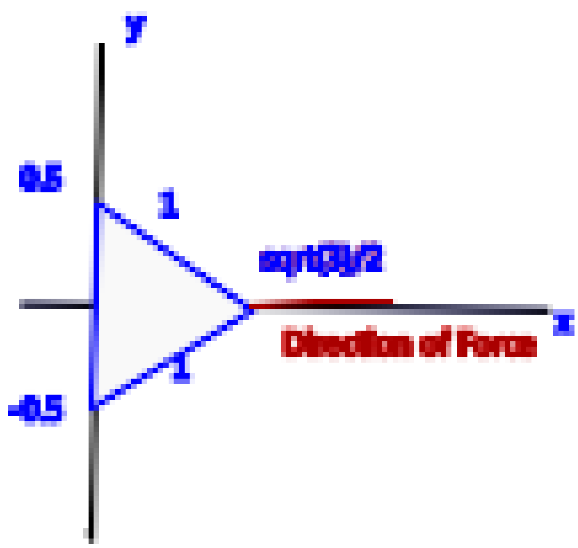
ASSIGNMENT 1

1. Three Electrong are placed at the corners of a equilateral triangle with sides length 1m, Find the net force on any one electior do to the other two

Let us consider the triange to be postioned as so:
$\sqrt{1-0.5^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$



Direction is, by symmetry going to be be away from the corner. So for the point on the x axis, the force will be along the x axis.

$|F| = \frac{1}{4\pi\epsilon_0} q_e(\frac{q_e}{1^2} + \frac{q_e}{1^2}) = \frac{q_e^2}{2\pi\epsilon_0} = 4.61 \times 10^{-28} N$

2. Find the work done of a charge of qC, by field $\mathbf{E} = (0, E)$in moving it along the quarter circle from $(5, 0)$to $(0, 5)$

$x = 5\cos\theta,\ y = 5\sin\theta, \theta \in [0, \frac{\pi}{2}]$
$l(\theta) = (x, y) = (5\cos\theta, 5\sin\theta)$
$\frac{dl}{d\theta} = (-5\sin\theta, 5\cos\theta)$
$W_{AB} = -Q\int_A^B \mathbf{E}\cdot d\mathbf{l} = -q\int_0^{\frac{\pi}{2}} (0, E)\cdot(-5\sin\theta, 5\cos\theta)d\theta = -q\int_0^{\frac{\pi}{2}} 5E\cos\theta d\theta$
$W = -5Eq\int_0^{\frac{\pi}{2}} \cos\theta d\theta = -5qE[\sin\theta]_0^{\frac{\pi}{2}} = -5qE$ J

3. Which of the following can not be the force created by an electric field on a charge of 1C located at (x,y,z)?

To be an electric field (force) must be conservitive.
ie at all points
$\exists f : \mathbb{R}^3 \to \mathbb{R}^3$, $\mathbf{F} = \nabla f$
iecurl $\mathbf{F} = \mathbf{0}$

3.1. $F = (x^2, y^2, z^2)$ **Yes.** $\operatorname{curl} F = (0-0, 0-0, 0-0) = \mathbf{0}$
Thus conservitive thus it could be an electic field

3.2. $\mathbf{F} = (-y^3, x^3, 0)$ **No.** $\operatorname{curl} F = (0-0, 0-0, 3x^2 + 3y^2) \neq \mathbf{0}$
Not an electric field.
t

3.3. $\mathbf{F} = r\hat{\boldsymbol{\theta}}$**No.** $F = \nabla f$
$\frac{\partial f}{\partial r} = 0$
$\frac{\partial f}{\partial \theta} = r^2$
$\frac{\partial f}{\partial z} = 0$
no such function can exist.
$(\operatorname{curl} F)_z = \frac{1}{r}(\frac{\partial rF_\theta}{\partial r} - \frac{\partial F_r}{\partial \theta}) = \frac{1}{r}(2r - 0) = 2 \neq 0$
Not a conservitive field
no a electrioc field

3.4. $\mathbf{F} = z\hat{\mathbf{r}} + r\hat{\mathbf{z}}$**yes.**

3.4.1. $\frac{\partial F_Z}{\partial \theta} = r\frac{\partial F_\theta}{\partial z}$. $0 = 0$

3.4.2. $\frac{\partial F_r}{\partial z} = \frac{\partial F_z}{\partial r}$. $1 = 1$

3.4.3. *Thus.* Curl is zero, so is conservitive,
so could be electric field

3.4.4. $\frac{\partial rF_\theta}{\partial r} = \frac{\partial F_r}{\partial \theta}$. $0 = 0$

3.4.5. *Thus.* Curl is zero, so is conservitive,
so could be electric field

3.5. $F = r\cos\phi\hat{\boldsymbol{\theta}}$**No.**

3.5.1. $\frac{\partial \sin\theta F_\phi}{\partial \theta} = \frac{\partial F_\theta}{\partial \phi}$. $0 \neq -r\sin\phi$
So curl $F \neq 0$
thus not conservitive
so not electric field

3.6. $F = \frac{\hat{\mathbf{r}}}{r^2} + \frac{1}{r}\hat{\boldsymbol{\theta}}$**Yes.**

3.6.1. $\frac{\partial \sin\theta F_\phi}{\partial \theta} = \frac{\partial F_\theta}{\partial \phi}$. 0=0

3.6.2. $\frac{\partial F_r}{\partial \theta} = \sin\theta \frac{\partial r F_\phi}{\partial r}$. $0 = 0$

3.6.3. $\frac{\partial r F_\theta}{\partial r} = \frac{\partial F_r}{\partial \theta}$. $0 = 0$

3.6.4. *Thus.* Curl is zero, so is conservitive,
so could be electric field

4. A charge of $-qC$ is placed in the center of a spherical, vacumn, caverty radius R, inside a perfect conductor. What is the charge induced on the surface of the caverty?

This is a boundry condtion problem.

4.1. **Electric Field Due to charge almost at boundry of vacum.** by guaus
Let us create a gausian sphere of radius:$R - \Delta h$,
$\oiint E \cdot dA = \text{flux} = \frac{Q_{enclosed}}{\epsilon_0}$
since perpendicular:
$\oiint E \cdot dA = EA = 4\pi(R - \Delta h)^2 E$
$4\pi(R - \Delta h)^2 E = \frac{Q_{enclosed}}{\epsilon_0}$
$E = \frac{-q}{\epsilon_0 4\pi(R - \Delta h)^2}$

4.1.1. *Consider a surface inside the conductor.* Let us conside a closed surface, with one side being parrell to the spherical cavert at $R+\Delta h$ distance from center.

4.1.2. *Let use now consider a Gaussian surface, crossing the boundary,* Lets make it a cut off spherical wedge. of height $2\Delta h$
such that: the inner surface has area: $A_i = k4\pi(R - \Delta h)^2$
and the outer surface: $k4\pi(R + \Delta h)^2$
for some constant $k < 1$
This surface will capture a portion of the charge induced on the inner surface of the conductor.
by symmetry it will capture $kQ_{inducted}$
There is a field: $D_i$though the inside edge,
since this boundry is shared with the sphere from 4.1, $D_i = \frac{-q}{\epsilon_0 4\pi(R - \Delta h)^2}$

4.1.3. *Apply Gauses law to spherical wedge.* Guas' law: $\oiint D \cdot dA = \text{flux} = \frac{Q_{enclosed}}{\epsilon_0}$

$\oiint D \cdot dA = D_i A_i + D_o A_o + D_{tangentaial} A_{side}$

with $A_{side} \propto \Delta h$ area of a cone slice

$\oiint D \cdot dA = D_i k 4\pi (R - \Delta h)^2 + D_o k 4\pi (R + \Delta h)^2 + D_{tangentaial} A_{side}$

consider $\Delta h \to 0$

$\oiint D \cdot dA = D_i A_i + D_o A_o + D_{tangentaial} A_{side} = k Q_{inducted}$

$D_i k 4\pi R^2 + D_o k 4\pi (R)^2 + 0 = k Q_{induced}$

$D_i + D_o = \frac{Q_{induced}}{4\pi R^2}$

$\frac{-q}{\epsilon_0 4\pi R^2} + D_o = \frac{Q_{induced}}{4\pi R^2}$

$Q_{induced} = 4\pi R^2 D_o - \frac{q}{\epsilon_0}$

Since D_o is the an electric field within a perfect conductor,

$D_o = 0$

Thus

$Q_{induced} = -\frac{q}{\epsilon_0}$

5. Sphere Radius R with charge density $\rho C/m^3$, test charge $-qC$ with mass m released from rest, at on one of narrow channel

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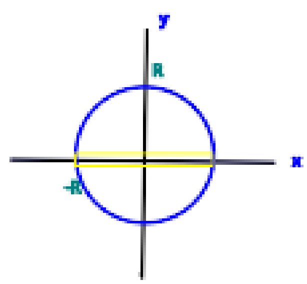

Let $p(t)$ be the position of the partical at time t

$p(0) = (-R, 0, 0)$

$p'(0) = \mathbf{0}$

$p''(t) = \frac{F(p(t))}{m}$

More over, since by symmetry, the partical is constrained to the x axis, we can simplify to 1 dimentional cordinates.

$p(0) = -R$

$p'(0) = 0$

$p''(t) = \frac{F(p(t))}{m}$

We must find the values of the Electric Field, along the x axis ($\phi = 0$, $\theta = \frac{\pi}{2}$ axis)

5.1. **Sphere Electric Field Values.**

5.1.1. *Use Gaus' Law inside.* Gaus law: $\oiint E \cdot dA = \text{flux} = \frac{Q_{enclosed}}{\epsilon_0}$

Lets consider, a point on the x Axis, that is on a gausian surface of radius T from center, $T \leq R$

$Q_{enc}(T) = V\rho = \frac{4}{3}\pi T^3 \rho$

By semetry we know that the field will be radial - perpendicular though the surface.

We also know, by syemtry that the field will be equal all all points on the surface.

So $\oiint_T E \cdot dA = E(T)A(T) = 4\pi T^2 E(T)$

thus

$\oiint E \cdot dA = \frac{Q_{enclosed}}{\epsilon_0}$

$4\pi T^2 E(T) = \frac{4\pi T^3 \rho}{3\epsilon_0}$

$E(T) = \frac{T\rho}{3\epsilon_0}$

Notice that there is a point in the center: $E(0) = 0$

5.1.2. *Use Gauses law Outside.* $Q_{enc}(R) = V\rho = \frac{4}{3}\pi R^3 \rho$

$\oiint E \cdot dA = \frac{Q_{enclosed}}{\epsilon_0}$

$4\pi T^2 E(T) = \frac{4\pi R^3 \rho}{3\epsilon_0}$

$E(T) = \frac{R^3 \rho}{3T^2 \epsilon_0}$

5.2. **Force.** $F(T) = -qE(T) = \begin{cases} \frac{-qT\rho}{3\epsilon_0} & T \leq R \\ \frac{-qR^3\rho}{3T^2\epsilon_0} & T > R \end{cases}$

Note that $q$is of different charge to the sphere,
so it will be attracted

5.3. **Work out position.** $p(0) = -R$

$p'(0) = \mathbf{0}$

$p''(t) = \frac{F(p(t))}{m} = \begin{cases} \frac{-|p(t)|q\rho}{3m\epsilon_0} & |p(t)| \leq R \\ \frac{-qR^3\rho}{3m|p(t)|^2\epsilon_0} & |p(t)| > R \end{cases}$

5.3.1. *Let us consider the case*$-R \leq p(t) \leq 0$*.* $p''(t) = -\frac{q\rho}{3m\epsilon_0}p(t)$

This a a 2ODE, solve it with Mathimatica.

$p(t) = -R\cos\left(t\sqrt{\frac{q\rho}{3m\epsilon_0}}\right)$

So we are oscillating with constant amplitude.
Worryingly, we have perpetural motion,
however this may be a consiquence of ignoring friction.
check: $p(0) = -R \times 1$

5.3.2. *Solve for other side:* $0 \leq p(t) \leq R$*.* When will we reach the center?

$\cos\left(t\sqrt{\frac{q\rho}{3m\epsilon_0}}\right) = 0 \iff t = \frac{\pi}{2\sqrt{\frac{q\rho}{3m\epsilon_0}}} \times k,\ k \in (2\mathbb{Z}+1)$

Giving us initiual contdion: $p(\frac{\pi}{2\sqrt{\frac{q\rho}{3m\epsilon_0}}}) = 0$

$p'(t) = R\sqrt{\frac{q\rho}{3m\epsilon_0}}\sin\left(t\sqrt{\frac{q\rho}{3m\epsilon_0}}\right)$

$p'(\frac{\pi}{2\sqrt{\frac{q\rho}{3m\epsilon_0}}}) = R\sqrt{\frac{q\rho}{3m\epsilon_0}}\sin\left(\frac{\pi}{2}\right) = R\sqrt{\frac{q\rho}{3m\epsilon_0}}$

$p''(t) = \frac{q\rho}{3m\epsilon_0}p(t)$

Could continue to solve this with mathimatica,
but this is getting convoluted, and error prone.

Assume that $p(t) = -R\cos\left(t\sqrt{\frac{q\rho}{3m\epsilon_0}}\right)$ holds for whole domain

5.3.3. *For it to return to start position.* $t\sqrt{\frac{q\rho}{3m\epsilon_0}} = 2\pi k,\ k \in \mathbb{Z}$

$t = \frac{2\pi k,}{\sqrt{\frac{q\rho}{3m\epsilon_0}}}\ k \in \mathbb{Z}$

Final Positon. No final position as constant motion.