Electromagnetic Theory ELEC 3303
Tutorial Assignment 2/Tutorial 11

22/23 May 2013

To be handed in by 5 p.m. on Monday 20 May 2013.

Please complete a cover sheet and attach it to your work.
Work handed in late will incur a penalty of 10% per day.
Work handed in after noon on 22 May will not be marked.

(1) A thin spherical shell made of non-conducting plastic has radius of 10 m. A hole whose area is 10^{-6} m^2 is punched in the shell. If the shell carries a surface charge distribution with density 1 C/m^2, estimate the electric field (magnitude and direction) at the centre of the shell if the shell is surrounded (inside and outside) by a vacuum. (**10 marks**)

(2)(a) A cube of side s m has its edges parallel to the $x-$, $y-$ and $z-$axes of a rectangular coordinate system. A uniform electric field $\boldsymbol{E}$ is parallel to the $x-$axis and a uniform magnetic field $\boldsymbol{B}$ is parallel to the $y-$axis. Use the Poynting vector to calculate the rate at which energy passes through each face of the cube. (**10 marks**)
(b) At what rate does the energy stored in the cube change? (**5 marks**)

(3)(a) A plane electromagnetic wave whose frequency is 1 MHz travels through a vacuum. The amplitude of the electric field of the wave is 10^3 V/m. Write down expressions for the electric and magnetic fields. (**15 marks**)
(b) What is the maximum instantaneous rate of energy propagation per square metre by the wave? (**5 marks**)
(c) Would the same amount of energy be transmitted by a wave whose frequency is 100 MHz? (**5 marks**)

(4) A plane wave with electric field strength

$$\boldsymbol{E}(x, y, z, t) = E_{i0}\sin(\omega t - \beta y)\hat{\boldsymbol{k}} \tag{1}$$

where $E_{i0} = 100\text{V/m}$ and $\beta = 4.189 \times 10^{-2}$ crosses the boundary from medium 1 to medium 2 to at $y = 0$. For medium 1, $\mu_1 = 4\mu_0$, $\varepsilon_1 = \varepsilon_0$, $\sigma_1 = 0\text{S/m}$. For medium 2, $\mu_2 = \mu_0$, $\varepsilon_2 = \varepsilon_0$, $\sigma_2 = 0\text{S/m}$. Find the Poynting vectors of the incident, transmitted and reflected waves. (**40 marks**)

(5) Two long thin parallel conductors separated by a distance dm and carrying currents I_1A and I_2A experience a force per unit length given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi d}. \tag{2}$$

The force is towards the other conductor if the currents flow in the same direction and away from the other conductor if the currents flow in opposite directions. Use this information and the Method of Images to find the force per unit length (magnitude and direction) of a wire carrying a current of 1A suspended 1m above an infinite perfectly conducting plane. (**10 marks**)

ATTACHMENT TO FINAL EXAMINATION

$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 Nm^2/C^2$. $\mu_0 = 4\pi \times 10^{-7} Wb/Am$.
Charge on an electron $e = -1.6 \times 10^{-19}C$. Mass of an electron $m_e = 9.11 \times 10^{-31}kg$.
Mass of a proton $m_p = 1.67 \times 10^{-27}kg$.
Dot product: $(v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}) \cdot (w_x\hat{\boldsymbol{\imath}} + w_y\hat{\boldsymbol{\jmath}} + w_z\hat{\boldsymbol{k}}) = v_xw_x + v_yw_y + v_zw_z$.
Cross product:
$(v_x\hat{\boldsymbol{\imath}}+v_y\hat{\boldsymbol{\jmath}}+v_z\hat{\boldsymbol{k}})\times(w_x\hat{\boldsymbol{\imath}}+w_y\hat{\boldsymbol{\jmath}}+w_z\hat{\boldsymbol{k}}) = (v_yw_z-v_zw_y)\hat{\boldsymbol{\imath}}+(v_zw_x-v_xw_z)\hat{\boldsymbol{\jmath}}+(v_xw_y-v_yw_x)\hat{\boldsymbol{k}}$.
Gradient of a scalar function f:

$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{k}}. \tag{3}$$

Divergence of a vector field $\boldsymbol{V}$:

$$\nabla \cdot \boldsymbol{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}. \tag{4}$$

Curl of a vector field $\boldsymbol{V}$:

$$\nabla \times \boldsymbol{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{\boldsymbol{k}}. \tag{5}$$

Flux of a vector field $\boldsymbol{F}$ through a surface S: $\Psi = \int\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$.
Gauss' Theorem: If F is a vector field and the closed surface S encloses a volume V,

$$\int\int_S \boldsymbol{F} \cdot d\boldsymbol{S} = \int\int\int_V \nabla \cdot \boldsymbol{F} dV. \tag{6}$$

The electric fields: $\boldsymbol{D} = \varepsilon\boldsymbol{E}$. The magnetic fields: $\boldsymbol{B} = \mu\boldsymbol{H}$.
The scalar potential: $\boldsymbol{E} = -\nabla V$. The vector potential: $\boldsymbol{B} = \nabla \times \boldsymbol{A}$.
Coulomb's Law:

$$|\boldsymbol{F}| = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R^2} \tag{7}$$

$$\boldsymbol{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\boldsymbol{r} - \boldsymbol{r}_k)}{|\boldsymbol{r} - \boldsymbol{r}_k|^3}. \tag{8}$$

Gauss' Law: $\nabla \cdot \boldsymbol{D} = \rho$. Divergence of the magnetic field: $\nabla \cdot \boldsymbol{B} = 0$.
Magnetic moment of a current loop: $\boldsymbol{m} = I\boldsymbol{A}$; I is current, $\boldsymbol{A}$ is area of loop.
The torque on a magnetic dipole with magnetic moment $\boldsymbol{m}$ in a magnetic field $\boldsymbol{B}$ is $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$.
Faraday's law of induction:

$$V_{emf} = \oint_L \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Psi}{dt} = -\frac{d}{dt}\int\int_S \boldsymbol{B} \cdot d\boldsymbol{S}. \tag{9}$$

Faraday's Law:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}. \tag{10}$$

Ampere's Law:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}. \tag{11}$$

Lorentz force law: $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$. The Poynting vector: $\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H}$.
Energy density of the electromagnetic field: $w_{EM} = \frac{\varepsilon}{2}\boldsymbol{E} \cdot \boldsymbol{E} + \frac{1}{2\mu}\boldsymbol{B} \cdot \boldsymbol{B}$.
Plane wave reflection coefficient: $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$.
Plane wave transmission coefficient: $\tau = \frac{2\eta_2}{\eta_2+\eta_1}$.