ASSIGNMENT 2

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1. Question 1

$R_{shell} = 10m$, $A_{shell} = 4\pi R_{shell}^2 = 400\pi$
$A_{hole} = 10^{-6}m^2$, assume spherical.
Consider the shell in polar coordinates:
$S : r = 10, \theta \in [0, \theta_{max}], \phi \in [0, 2\pi]$
what proportion missing?
$\theta_{max} = \pi - \theta_{hole}$
Area is proportial to the range of θ, $\frac{A_{shell}}{A_{hole}} = \frac{400\pi}{10^{-6}} = \frac{\pi}{\theta_{hole}}$
$\theta_{hole} = \frac{10^{-6}}{400} = 0.25 \times 10^{-8}$
$\theta_{max} = \pi - \frac{10^{-8}}{4}$
$S : r = 10, \theta \in [0, \pi - \frac{10^{-8}}{4}], \phi \in [0, 2\pi]$

1.1. Find field.

$\rho = 1C/m^2$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_\theta \int_\phi \frac{\rho\left[\begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}r\\\theta\\\phi\end{pmatrix}\right]}{\left|\begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}r\\\theta\\\phi\end{pmatrix}\right|^3} r^2 \sin\theta d\phi d\theta$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{\theta\in[0,\pi-0.24\times10^{-8}]} \int_{\phi\in[0,2\pi]} \frac{\rho\left[-\begin{pmatrix}10\\\theta\\\phi\end{pmatrix}\right]}{10^3} 10^2 \sin\theta d\phi d\theta$$

By considering this eqation in vector form, and they symetries involved we can see that the electric field will be *pointing towards the hole.*

But lets simiplify it to find the magnitude

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{\theta=0}^{\pi-0.24\times10^{-8}} \int_{\phi\in0}^{2\pi} \frac{\rho\left[-\begin{pmatrix}10\\\theta\\\phi\end{pmatrix}\right]}{10} \sin\theta d\phi d\theta$$

Appears to be independent of the radius of the shell (other than the size of the hole compared to it)

$E = \frac{1}{4\pi\epsilon_0} \int_{\theta=0}^{\pi-0.24\times10^{-8}} \int_{\phi\in0}^{2\pi} \rho \sin\theta d\phi d\theta$
$E = \frac{1}{4\pi\epsilon_0} \int_{\theta=0}^{\pi-0.24\times10^{-8}} \int_{\phi\in0}^{2\pi} \sin\theta d\phi d\theta$
$E = \frac{1}{2\epsilon_0} \int_{\theta=0}^{\pi-0.24\times10^{-8}} \sin\theta d\theta = \frac{1}{2\epsilon_0} [-\cos\theta]_0^{\pi-0.24\times10^{-8}}$
$E = \frac{1}{2\epsilon_0}(1+1) = \frac{1}{\epsilon_0}$

The field has magnitude $\frac{1}{\epsilon_0} = 1.13 \times 10^{11} N/C$, and is pointing in the direction of the hole.

2. Cube

2.1. **a).** $\mathbf{E} = \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix}$, $\mathbf{S} = \mathbf{E} \times \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ EB \end{pmatrix} = EB\hat{\mathbf{k}}$ -> in the Z direction

2.1.1. *bottom/top (parallel to x,y axis).* Energy through face=s^2EB

2.1.2. *right/left (parallel to z,x axis).* Energy through face=0

2.1.3. *back/front (parallel to z,y axis).* Energy through face=0

2.2. **b) Rate of energy change.** $-\frac{dW}{dt} = -\frac{d}{dt}(\frac{B \cdot H + D \cdot E}{2}) = -J \cdot E + \nabla \cdot (\mathbf{E} \times \mathbf{H})$
$J = 0$
$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \nabla \cdot S = 0 + 0 + \frac{d}{dz}(EB) = 0$ as E and B are uniform.
Rate of energy change:$-\frac{\partial W}{\partial t} = 0$

3. Question 3 Plane Waves

3.1. **A).** Assume WLOG, that the electic field is in the $\hat{\mathbf{k}}$ direction and the wave is propergatin in the $\hat{\mathbf{i}}$ direction, thus the magnetic field is the the $\hat{\mathbf{j}}$ direction
$f = 10^6 Hz$, in a vacum, $A = 10^3 V/m$ - no attenuation in a vacuum
$w = 2\pi f = 2\pi \times 10^6$
$\beta = \frac{w}{c} = \frac{2\pi \times 10^6}{3 \times 10^8} = \frac{2\pi}{300}$
$\mathbf{E} = Acos(wt - \beta x)\hat{\mathbf{k}} = 1000\cos(2\pi \times 10^6 t - \frac{2\pi}{300}x)\hat{\mathbf{k}}$
$\eta_0 = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}$
$\mathbf{H} = \frac{1000}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\cos(2\pi \times 10^6 t - \frac{2\pi}{300}x)\hat{\mathbf{j}}$

3.2. **Find maximum instantataions power.** Poynting vector $\mathbf{S}$
$\mathbf{S} = \mathbf{E} \times \mathbf{H} = 1000\cos(2\pi \times 10^6 t - \frac{2\pi}{300}x)\hat{\mathbf{k}} \times \frac{1000}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\cos(2\pi \times 10^6 t - \frac{2\pi}{300}x)\hat{\mathbf{j}}$
$\mathbf{S} = \frac{10^6}{\sqrt{\frac{\mu_0}{\epsilon_0}}}\left(\cos(2\pi \times 10^6 t - \frac{2\pi}{300}x)\right)^2 \hat{\mathbf{i}}$
$\mathbf{S}_{max} = \frac{10^6}{\sqrt{\frac{\mu_0}{\epsilon_0}}} W/m^2 = 10^6\sqrt{\frac{\epsilon_0}{\mu_0}}\, J/sm^2$

3.3. **Would the same amount of energy be transferred if the freqency was $100MHz$.** The same maximum instantatious power would occur, as this is frequency independent.
and indeed the total energy transferred would be the same.

4. Plane wave crossing boundry

$\mathbf{E} = E_{i0}sin(wt - \beta y)\hat{\mathbf{k}}$
$E_{io} = 100V/m$
$\beta_i = 4.189 \times 10^{-2}$
Boundry is at $y = 0$

4.0.1. *Parameters.*

$\mu_1 = 4\mu_0$	$\mu_2 = \mu_0$
$\epsilon_1 = \epsilon_0$	$\epsilon_2 = \epsilon_0$
$\sigma_1 = 0$	$\sigma_2 = 0$
$\eta_1 = 2\sqrt{\frac{\mu_0}{\epsilon_0}}$	$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

4.0.2. *Find angular frequency.* $w = \frac{\beta_i}{\sqrt{u_i\epsilon_1}} = \frac{4.189}{200\sqrt{\mu_0\epsilon_0}}$

4.1. **Incident Wave.**

4.1.1. *Incident Electric field.* $\mathbf{E}_i = 100sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{100}y)\hat{\mathbf{k}}$

4.1.2. *Incident Magetic field.* Wave is moiving in the $\hat{\mathbf{j}}$direction, electric field in the $\hat{\mathbf{k}}$direction, thus magnetic field is the the $\hat{\mathbf{i}}$direction.
$H_{0i} = \frac{E_{0i}}{\eta_1} = 50\sqrt{\frac{\epsilon_0}{\mu_0}}$
$\mathbf{H}_i = 50\sqrt{\frac{\epsilon_0}{\mu_0}}sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{100}y)\hat{\mathbf{i}}$

4.1.3. *Incident Poynting Vector.* $\mathbf{S_i} = \mathbf{E}_i \times \mathbf{H}_i = 5000\sqrt{\frac{\epsilon_0}{\mu_0}}sin^2(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{100}y)\hat{\mathbf{j}}$

4.2. **Reflected Wave.** Wave is now propegating in $-\hat{\mathbf{j}}$ direction.
Same medium so same β, but opposite sign

4.2.1. *Reflection coeffient.* $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \dfrac{\sqrt{\frac{\mu_0}{\epsilon_0}} - 2\sqrt{\frac{\mu_0}{\epsilon_0}}}{3\sqrt{\frac{\mu_0}{\epsilon_0}}} = \dfrac{-1}{3}$

We have has a phase flip.

4.2.2. *Reflected Electric field.* $E_{R0} = \Gamma E_{io} = \frac{-100}{3}$
$\mathbf{E}_i = \frac{-100}{3}sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t + \frac{4.189}{100}y)\hat{\mathbf{k}}$

4.2.3. *Reflected Magetic field.* Wave is moiving in the $-\hat{\mathbf{j}}$direction, electric field in the $-\hat{\mathbf{k}}$direction, thus magnetic field is (still) the the $\hat{\mathbf{i}}$direction.
$H_{0r} = \frac{E_{0r}}{\eta_1} = \dfrac{100}{3 \times 2}\sqrt{\dfrac{\epsilon_0}{\mu_0}}$
$\mathbf{H}_r = \dfrac{50}{3}\sqrt{\dfrac{\epsilon_0}{\mu_0}}\,\sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t + \frac{4.189}{100}y)\hat{\mathbf{i}}$

4.2.4. *Reflected Poynting Vector.* $\mathbf{S}_r = \mathbf{E}_r \times \mathbf{H}_r = -\dfrac{5000}{9}\sqrt{\dfrac{\epsilon_0}{\mu_0}}\sin^2\left(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t + \frac{4.189}{100}y\right)\hat{\mathbf{j}} = -\Gamma^2\mathbf{S}_i = -\dfrac{1}{9}\mathbf{S}_i$

4.3. **Transmitted Wave.** Wave is still propegating in $\hat{\mathbf{j}}$ direction.
different medium so different β, but same sign
Frequency is constant because that is set by the source
$w = \frac{\beta_i}{\sqrt{u_i\epsilon_i}} = \frac{\beta_r}{\sqrt{u_t\epsilon_t}} = \frac{4.189}{200}\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}$
$\frac{4.189}{100} \times \frac{1}{2\sqrt{\mu_0\epsilon_0}} = \beta_2 \times \frac{1}{\sqrt{\mu_0\epsilon_0}}$
$\beta_2 = \frac{4.189}{200}$

4.3.1. *transmission coeffient.* $\tau = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{2\sqrt{\frac{\mu_0}{\epsilon_0}}}{3\sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{2}{3}$

4.3.2. *Transmitted Electric field.* $E_{t0} = \tau E_{io} = \frac{200}{3}$
$\mathbf{E}_t = \frac{200}{3} sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{200}y)\hat{\mathbf{k}}$

4.3.3. *Transmitted Magetic field.* Wave is moving in the $\hat{\mathbf{j}}$direction, electric field in the $\hat{\mathbf{k}}$direction, thus magnetic field is (still) the the $\hat{\mathbf{i}}$direction.
$H_{t0} = \frac{E_{ot}}{\eta_2} = \frac{200}{3}\sqrt{\frac{\epsilon_0}{\mu_0}}$
$\mathbf{H}_t = \frac{200}{3}\sqrt{\frac{\epsilon_0}{\mu_0}} sin(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{200}y)\hat{\mathbf{i}}$

4.3.4. *Transmitted Poynting Vector.* $\mathbf{S}_t = \mathbf{E}_t \times \mathbf{H}_t = \frac{40000}{9}\sqrt{\frac{\epsilon_0}{\mu_0}} \sin^2\left(\frac{4.189}{200\sqrt{\mu_0\epsilon_0}}t - \frac{4.189}{200}y\right)\hat{\mathbf{j}}$

5. Method Of Images

In the reflection each charged partial is replaced with a particle of opposite charge.

When this partical moves parell to the equiptenail plane, so does its negitive reflection (in the smae direction).

So the relected current flows in the opposite direction. So this means the wire is attacted to its reflection (and thus in reality to the perfectly conducting plate.

$F = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{\mu_0 \times 1 \times 1}{2\pi \times (1+1)} = \frac{\mu_0}{4\pi}$